

From the Editor

'I love numbers'

What do you say in a five-minute slot to a nonmathematical audience with ages ranging from 5 to 95? It was the beginning of January, so we could start by factorizing the year:

$$2016 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7.$$

A quick look back to the previous year gives

$$2015 = 5 \times 13 \times 31,$$

and a sneak look ahead gives 2017, which cannot be written as a product of smaller numbers; it is prime.

I was asked afterwards if 2019 is prime; the questioner realized that 2018 is divisible by 2. I was able to describe a test for a number to be divisible by 3, namely: is the sum of its digits divisible by 3? Since $2 + 0 + 1 + 9 = 12$, which is divisible by 3, then 2019 is also divisible by 3, so is not prime. The question was then asked: what is the next prime after 2017? I had to consult my list of primes up to 5000 to find out that the next two primes are 2027 and 2029, a 'prime pair', i.e. two primes only two apart, and was able to tell the questioner that, although there are infinitely many primes, it is not known whether there are infinitely many prime pairs.

Most of the audience knew that 2016 is a leap year, because 2016 is divisible by 4. Thus it has an extra day, or 24 hours, or 1440 minutes, or 86 400 seconds. What shall we do with all these extra seconds, we asked. Betty was born on 29 February, so although she is nearly 84 years old, she has only had 20 birthdays, a reminder of the paradox in Gilbert and Sullivan's *The Pirates of Penzance*. The children in the audience were full of sympathy for Betty's plight, but we cheered at the thought that Betty is about to have her 21st birthday.

Had there been time, we could have gone on to ask whether our year is the sum of two squares. We might have discovered that

$$2017 = 9^2 + 44^2$$

but that neither 2015 nor 2016 is a sum of two squares. In fact, a number is the sum of two squares if and only if all its prime factors of the form $4k + 3$ occur an even number of times in its prime factorization. Thus, the prime factors 7 of 2016 and 31 of 2015, both of the form $4k + 3$ and occurring only once in their prime factorizations (and 1 is odd), stop 2016 and 2015 being sums of two squares. On the other hand, 2017 has no prime factors of the form $4k + 3$ so, by default, 2017 satisfies the condition to be a sum of two squares.

Perhaps we should stop whilst we are ahead. After all, the aim was to stimulate a love of numbers, not to put the audience off.