Published by the **Applied Probability Trust**

Computing Geographical Distances; Derivation and Application

CHASE ASHBY and JOSH THOMPSON

We explain two existing methods for computing distances on Earth, one using great circles (orthodromes) and the other using great spirals (loxodromes). We derive formulas for these approaches, and use each of them to calculate the shoreline distances around each of the Great Lakes. The resulting calculations compare favourably with established statistics of the Great Lakes.

Introduction

In Euclidean geometry two points on a plane determine a unique straight line passing through them. In spherical geometry the notion of 'straight' can be interpreted in multiple ways. For example, a straight line connecting two points on a sphere could refer to the arc connecting them which is contained in a plane passing through the centre of the sphere. Alternatively, a straight line connecting these points could be defined as the path connecting the points which takes a constant bearing from one point to the other. These different approaches give different ways to measure distance on a sphere. We derive two methods for computing distance based on these ideas, apply them both to real data, and compare the results.

Loxodromes

A *loxodrome* is a path spiralling from the North Pole to South Pole that crosses each meridian (a great circle passing through the North Pole and South Pole) at the same angle. Two arbitrary geographic points are connected by many different loxodromic paths. There is, however, a unique shortest path corresponding to the path that spirals the least as it makes its way from point to point. In this method, we define the distance between two points on Earth to be the length of the shortest loxodromic path connecting the two points. To find this length we first use a Mercator projection to map the spiralling path into a flat plane. Then we take the standard Euclidean distance of the transformed path and define that to be the loxodromic distance between the two points on Earth. The Mercator map is a conformal cylindrical projection which preserves angles and maps lines of constant latitude to horizontal lines. Since angles are preserved under this mapping meridians are mapped to vertical lines. The Mercator projection assumes a spherical Earth and we make the same assumption.

We begin by writing parametric equations describing the line between two arbitrary locations on the Mercator projection. Because the projection is invertible, these equations will allow us to write parametric equations describing the loxodromic path between the same two points on Earth.

Let p denote a parallel of latitude (a great circle perpendicular to every meridian) north of the equator of radius r. Note that p makes an angle ϕ with the equator as shown in figure 1.