Published by the **Applied Probability Trust**

Complex Numbers and Desargues' Theorem

GUIDO LASTERS and DAVID SHARPE

We show how Desargues' theorem in geometry can be proved using complex numbers and illustrate how a multiple use of Desargues' theorem can be used to prove a result in geometry.

In our companion article in this issue 'Van Aubel's theorem using complex numbers' we showed how complex numbers can be used to give a neat proof of Van Aubel's theorem in geometry. This spurred us on to try to do a similar thing with the better-known Desargues' theorem. According to Wikipedia, Desargues did not publish his theorem but it was in an appendix, entitled "Universal method of M. Desargues' for using perspectives", of a book on perspective by a pupil of his, Abraham Bosse, in 1648.

Desargues' theorem is illustrated in figure 1. Triangles ABC and $A'B'C'$ are such that AA', BB', CC' meet at a point P, say. We say that they are *in perspective* from P and call P their *point of perspectivity*. The sides BC and $B'C'$ are produced to meet at X, CA and $C'A'$ are produced to meet at Y, and AB and $A'B'$ are produced to meet at Z. Desargues' theorem says that X, Y, Z are collinear. We call X, Y, Z their *axis of perpectivity*. A beautiful theorem, we hope readers will agree.

We now show how this may be proved using complex numbers. We choose P as the origin of the Argand diagram, and denote by a, b, c the complex numbers representing A, B, C respectively, and by ka, lb, mc the complex numbers representing A', B', C' respectively, where k, l, m are real numbers. A point on BC will be represented by a complex number of the form $\lambda b + (1 - \lambda)c$ for some real number λ, and a point on $B'C'$ by $\mu lb + (1 - \mu)lc$ for

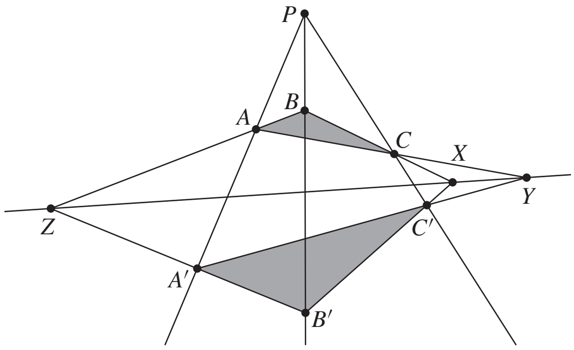

Figure 1

Accepted by C. Jordan, Pure Mathematics Editor.