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Van Aubel's Theorem using Complex Numbers

GUIDO LASTERS and DAVID SHARPE

We show how complex numbers can be used to give a neat proof of van Aubel's theorem and an extension of it, and leave readers with a challenge.

Van Aubel (1830–1906) was a teacher of mathematics in the Royal Atheneum in Antwerp. The theorem which bears his name goes back to 1878 (see reference 1). Start with any quadrilateral in the plane and construct squares on its four edges outside the quadrilateral. Join the centres of opposite squares to give two line segments. Then these two line segments are equal in length and at right angles (see figure 1).

Readers will find various proofs of this delightful result on the web (e.g. reference 2). In a recent article in *Mathematical Spectrum*, Paul Glaister gave a proof using vectors (see reference 3). In this article we will use complex numbers to prove van Aubel's theorem and also to prove an extension of it.

We consider the quadrilateral in the Argand diagram and denote by a, b, c, d the complex numbers which represent its vertices (see figure 1). The centre of the square on the edge ab is found by moving from the centre of that edge at right angles to it a distance of half the length

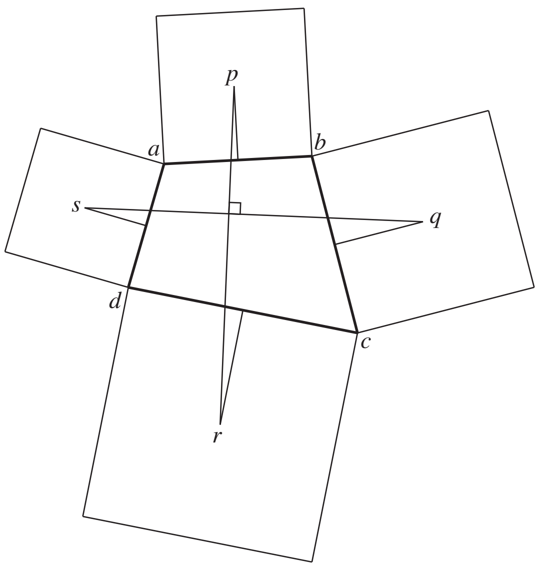

Figure 1

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